# The Madness of Multiple Entries in March Madness

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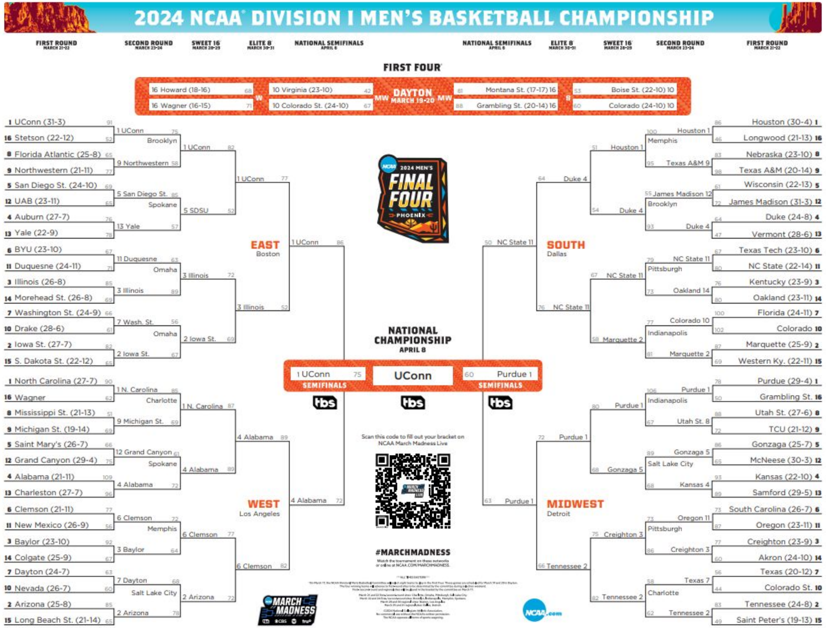
# March Madness

- College basketball end-of-season tournament
- 64-teams single-elimination tournament
  - 4 regions of 16 teams
  - Teams ranked from 1 to 16
- In 2023, AGA estimated that 68 million American spent $15.5 billion on March Madness[2]
- In 2014, Warren Buffet $1 billion for a perfect bracket[3]




March Madness Bracket 2024[1]

[1]NCAA(2024) - Latest bracket, schedule and scores for 2024 NCAA men's tournament. URL: https://www. ncaa.com/brackets/print/basketball-men/d1/2024
[2]American Gaming Association (2023) - 68 Million Americans to Wager on March Madness. URL: https://www.americangaming.org/new/68-million-americans-to-wager-on-march-madness/
[3]Forbes(2014) - Warren Buffett Offers $1 Billion for Perfect March Madness Bracket. URL: https://www.forbes.com/sites/kellyphillipserb/2014/01/21/warren-buffett-offers-1-billion-for-perfect-march-madness-bracket/

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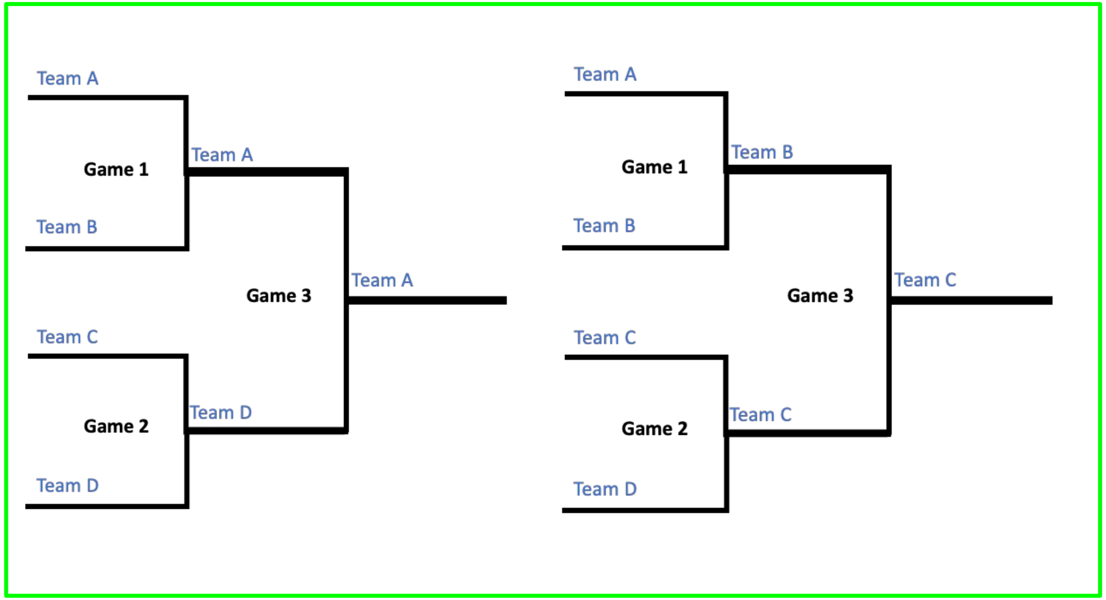
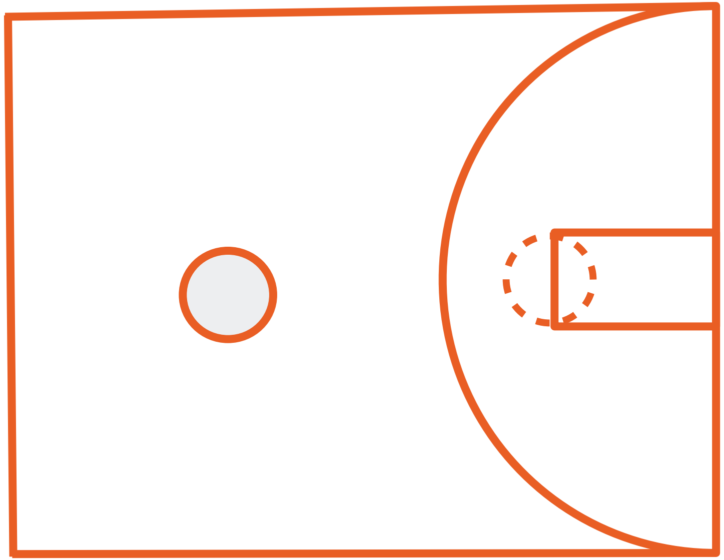


# Betting Pool

**Step 1: Enter Contest**

- Pay entry cost



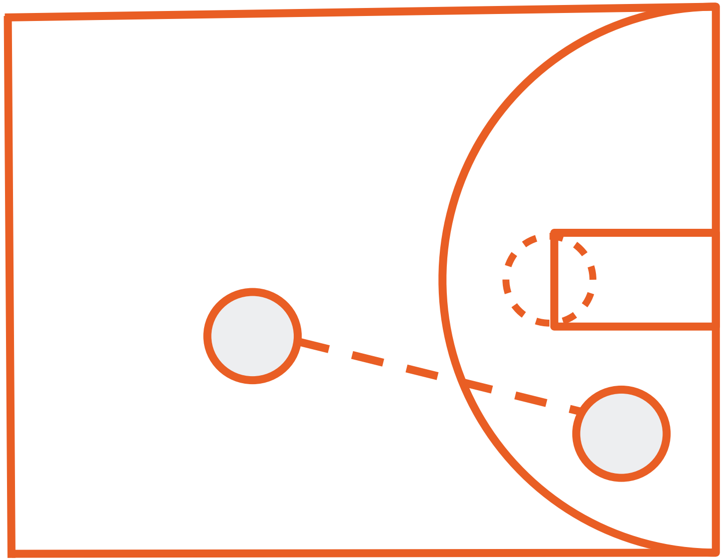
# Betting Pool

**Step 1: Enter Contest**

- Pay entry cost

**Step 2: Make Selections**

- Select the winner for all games in the tournament
- Selection must be consistent across round



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# Betting Pool

**Step 1: Enter Contest**

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**Step 2: Make Selections**

- Select the winner for all games in the tournament
- Selection must be consistent across round

**Step 3: Observe Outcomes**

- Score points for every correct prediction

| Round | Scoring System | |
|---|---|---|
| 1 | 1 point | $2^{\text{round}-1}$<br>Max Score 192 |
| 2 | 2 points | |
| 3 | 4 points | |
| 4 | 8 points | |
| 5 | 16 points | |
| 6 | 32 points | |

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**Step 4: Payoff Structure**

- Entries are ranked based on the score
- Receive payoff according to the ranking

| Rank | Payoff Structure |
|---|---|
| 1st | $1,000,000 (80%) |
| 2nd | $50,000 (4%) |
| 3rd | $20,000 (1.6%) |
| 4th | $10,000 (0.8%) |
| ... | .. |
| 1000th | $100 (0.012%) |

# Outline

- Literature review
- Strategy and research question
- Methodology
- Results
- Conclusion

# Literature Review - Single Entry Problems

- March Madness and the office pool[3]
  - Dynamic Programming algorithm that maximizes the expected score of a single entry
- Optimal Strategies for sports betting pool[4]
  - Crowd avoidance is often a more profitable strategy

3 Kaplan, E. H., & Garstka, S. J. (2001). March madness and the office pool. Management Science, 47(3), 369-382.

4 Clair, B., & Letscher, D. (2007). Optimal strategies for sports betting pools. Operations Research, 55(6), 1163-1177.

# Literature Review - Multiple Entries Problems

- Maximizing expected score
    - Surviving a National Football League survivor pool[5]:
    - Picking winners in daily fantasy sports using integer programming[6]
    - Optimizing the expected maximum of two linear functions defined on a multivariate Gaussian distribution[7]
    - Picking winners: Diversification through portfolio optimization[8]
- Maximizing Expected Payoff
    - How to play strategically in fantasy sports (and win)[9]

5 Bergman D., Imbrogno J. (2017) Surviving a National Football League survivor pool. Operations Research 65(5): 1343–1354.

6 Hunter, D. S., Vielma, J. P., & Zaman, T. (2016). Picking winners in daily fantasy sports using integer programming. *arXiv preprint arXiv:1604.01455*.

7 Bergman, D., Cardonha, C., Imbrogno, J., & Lozano, L. (2023). Optimizing the expected maximum of two linear functions defined on a multivariate Gaussian distribution. *INFORMS Journal on Computing*, *35*(2), 304-317.

8 Liu, J., Liu, C., & Teo, C. P. (2023). Picking winners: Diversification through portfolio optimization. *Production and Operations Management*.

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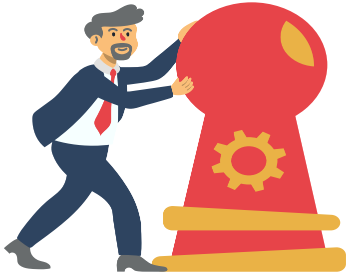

# Our Strategy

## Multiple Entries Betting Pools

Select an optimal collection of entries that maximizes the expected score of the maximum scoring entry

# Notations

- Let $\mathcal{B}$ be the collection of all feasible tournaments indexed by O

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- Let E $\in \{0,1\}^{|\mathscr{T}| x G}$ denote an entry with $E_{t,g}$=1 if and only if team t is selected to win game g
    a. Only 1 team chosen per game g
    b. A team must have been selected in every round prior to the round of game g

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  } Bracket Feasibility Constraints
- Let S(E) be the random variable representing the score of entry E
- Let S(E,O) be the score obtained by entry E when outcome O is observed
- Let $\mathcal{E}$ be the collection of entries

# Problem Parameters

Winning Probability Matrix: P
($P_{AB}=1-P_{BA}$)

| | A | B | C | D |
|---|---|---|---|---|
| A | - | $P^{team}_{A,B}$ | $P^{team}_{A,C}$ | $P^{team}_{A,D}$ |
| B | $P^{team}_{B,A}$ | - | $P^{team}_{B,C}$ | $P^{team}_{B,D}$ |
| C | $P^{team}_{C,A}$ | $P^{team}_{C,B}$ | - | $P^{team}_{C,D}$ |
| D | $P^{team}_{D,A}$ | $P^{team}_{D,B}$ | $P^{team}_{D,C}$ | - |

Bracket Structure


Points Structure

| Round | Scoring System |
|---|---|
| 1 | 1 point |
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# Our Strategy

## Multiple Entries Betting Pools

**Select an optimal collection of entries that maximizes the expected score of the maximum scoring entry**

$$S(\mathcal{E}) := \max_{E \in \mathcal{E}} S(\{E\})$$

$$\max_{\mathcal{E} \subseteq \mathcal{B} : |\mathcal{E}| = e} \mathbb{E}[S(\mathcal{E})]$$

## Research Question

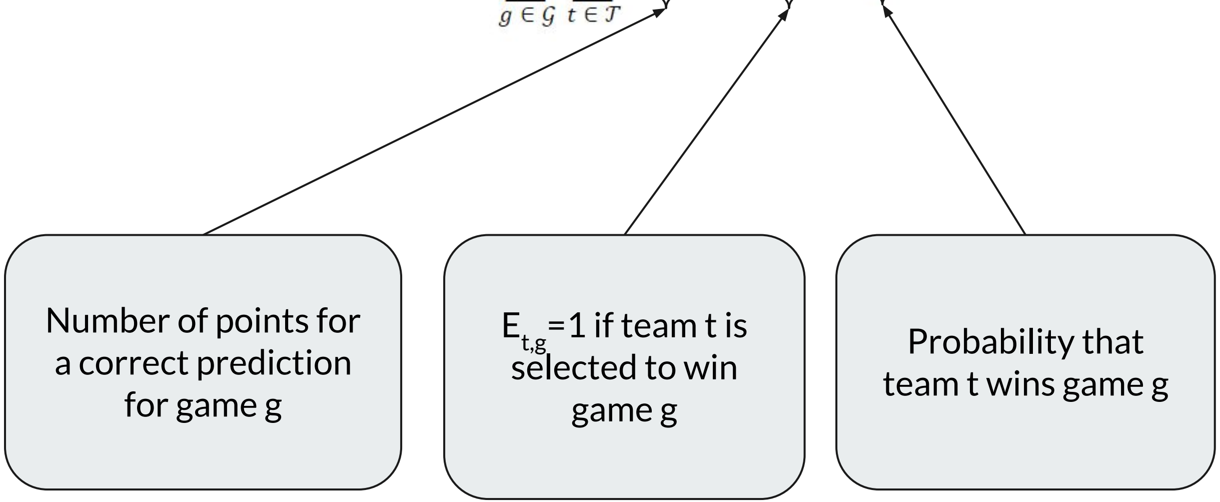
### Multiple Entries Betting Pools

Select an optimal collection of entries that maximizes the expected score of the maximum scoring entry

Research Question: How does this strategy compares in performance against the betting strategies employed by elite sports bettors?

# Calculating the Expected Value of Single Entry

- The expected score[1] of any single-entry E is: $\mathbb{E}[S(E)] := \sum_{g \in \mathcal{G}} \sum_{t \in \mathcal{T}} \underbrace{2^{r(g)-1}} \cdot \underbrace{E_{t,g}} \cdot \underbrace{P_{t,g}^{\text{game}}}$




[1]Kaplan, E. H., & Garstka, S. J. (2001). March madness and the office pool. Management Science, 47(3), 369-382.

# Calculating the Expected Value for Multiple Entries

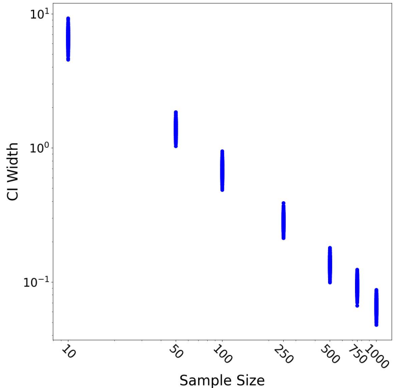
- The expected score of any two-entry $\mathcal{E} = \{E^1, E^2\}$ is: $\mathbb{E}[S(\mathcal{E})] = \sum_{O \in \mathcal{B}} P_O \max\left( s(E^1, O), s(E^2, O) \right)$

- **Theorem 1:** The expected score of the maximum scoring entry can be computed in time: $O\left(\mathrm{t} \cdot (\mathrm{t} \cdot \log_2(\mathrm{t}))^{2\mathrm{e}+1}\right)$

# Calculating the Expected Value
## Multiple Entries

- Sample Average Approximation (SAA)

$$\widehat{\mathbb{E}\left[S(\mathcal{E})\right]} = \frac{1}{|O_{\texttt{Sim}}|} \sum_{O \in O_{\texttt{Sim}}} \left( \max_{E \in \mathcal{E}} S(E, O) \right)$$




$|O_{sim}|$ = 250 Simulations is enough

# Structural Results for Multiple Entries Problem

- **Proposition 1:** The Function $\mathbb{E}[S(\mathcal{E})]$ is submodular function
- **Theorem 2:** With 0.5 probability for all matchups, any two disjoint brackets are optimal
- **Remark 1:** An entry with the highest single-entry expected score is not necessarily part of the optimal collection of two entries
- **Remark 2:** Optimal multiple entries may select the same winner of the tournament

# MIP Formulation for Single Entry Problem

$$
\begin{aligned}
\max \quad & \sum_{g \in \mathcal{G}} \sum_{t \in \mathcal{T}(g)} 2^{r(g)-1} \cdot x_{t,r(g)} \cdot P^{\text{round}}_{t,r(g)} & \\
\text{s.t.} \quad & \sum_{t \in \mathcal{T}(g)} x_{t,r(g)} = 1, & \forall g \in \mathcal{G} \\
& x_{t,r} \leq x_{t,r-1}, & \forall t \in \mathcal{T}, r \in \mathcal{R} \setminus \{1\} \\
& x_{t,r} \in \{0,1\}, & \forall t \in \mathcal{T}, r \in \mathcal{R}.
\end{aligned}
$$

Bracket Feasibility Constraints

# MIP Formulation for Multiple Entries Problem

$$\max \quad \frac{1}{|O_{\mathtt{Sim}}|} \sum_{O \in O_{\mathtt{Sim}}} \left( \max_{E \in \mathcal{E}} S(E, O) \right)$$

s.t.

Bracket Feasibility Constraints

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$$\max \quad \frac{1}{|O_{\texttt{Sim}}|} \sum_{O \in O_{\texttt{Sim}}} \left( \max_{E \in \mathcal{E}} S(E, O) \right)$$

s.t.

Bracket Feasibility Constraints

MIP

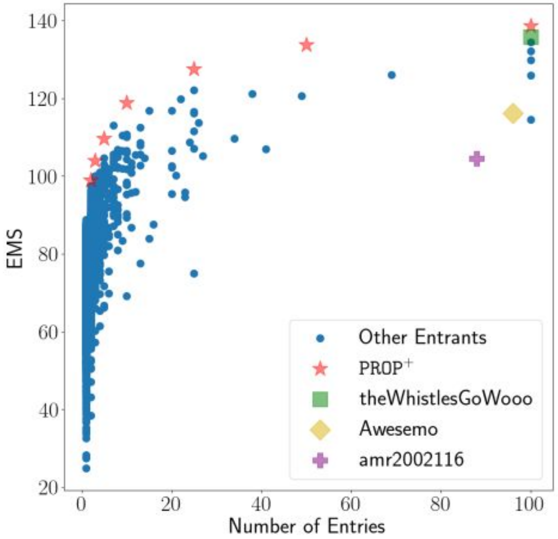
$$
\begin{aligned}
&\max \frac{1}{\mathrm{w}} \sum_{w=1}^{\mathrm{w}} s_w^{\max} && \\
&\sum_{t \in T(g)} x_{t,r(g),e} = 1 && \forall\, g \in \mathcal{G}, \forall\, e \in [\mathrm{e}] \\
&x_{t,r,e} \le x_{t,r-1,e} && \forall\, t \in \mathcal{T}, \forall\, r \in \mathcal{R} \setminus \{1\}, \forall\, e \in [\mathrm{e}] \\
&s_{w,e} = \sum_{g \in \mathcal{G}} 2^{r(g)-1} \cdot W_{t,g}^{w} \cdot x_{t,r(g),e} && \forall\, w \in [\mathrm{w}], \forall\, e \in [\mathrm{e}] \\
&\sum_{e \in [\mathrm{e}]} z_{w,e} = 1 && \forall\, w \in [\mathrm{w}] \\
&s_w^{\max} \le s_{w,e} z_{w,e} + M(1 - z_{w,e}) && \forall\, w \in [\mathrm{w}], \forall\, e \in [\mathrm{e}] \\
&x_{t,r,e} \in \{0, 1\} && \forall\, t \in \mathcal{T}, \forall\, r \in \mathcal{R}, \forall\, e \in [\mathrm{e}] \\
&s_{w,e}, s_w^{\max} \in \mathbb{R}_+ && \forall\, w \in [\mathrm{w}], \forall\, e \in [\mathrm{e}].
\end{aligned}
$$

# DraftKings Contest

- DraftKings Contest
- Only available for the best sport bettors
- $100/entry (max 100 entries)
- 10,000 randomly generated brackets are used to evaluate solutions
- $1,000,000 to the winner

| Entries | 1 | 2 | 3 | 4-5 | 6-10 | 11-25 | 26-50 | 51-99 | 100 | Total |
|---|---|---|---|---|---|---|---|---|---|---|
| Number of Contestants | 7,756 | 790 | 179 | 122 | 72 | 33 | 6 | 3 | 6 | 12,605 |

# DraftKings Contest (Using 538 Probability Matrix)




(a) Empirical EMS (538)

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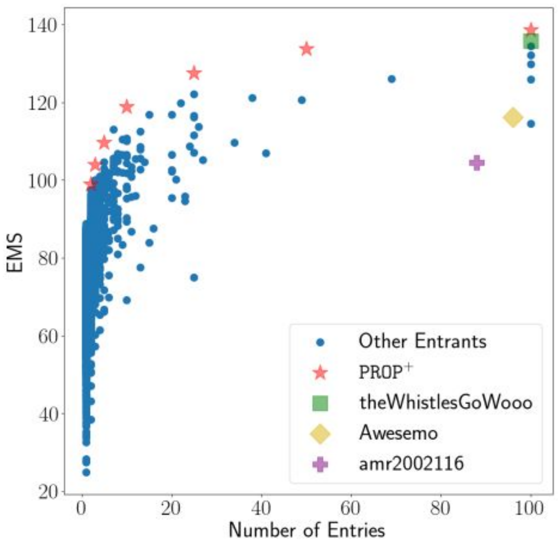


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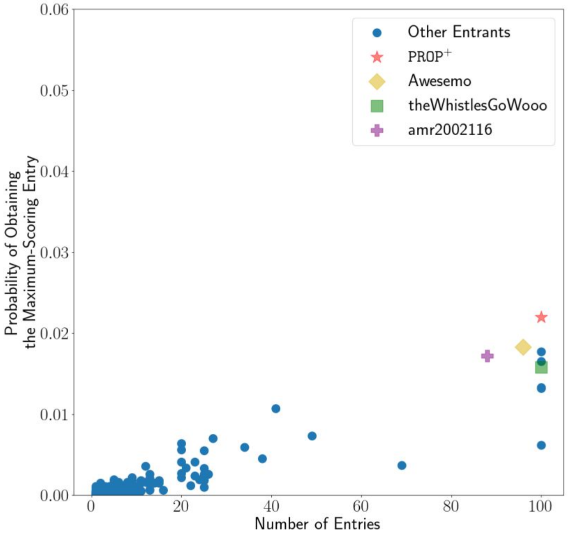


(b) Victory Probability (538)

# Conclusion

- March Madness Challenge
- Maximize the expected score of the maximum scoring entry
    - 250 simulations provide good estimate of the expected score
    - SAA is the best for a small number of entries
    - Structural results
    - Prop+ outperforms all algorithms for a large number of entries
- DraftKings contest
    - Highest expected score among all participants
    - 2.2% chance of winning using Prop+
    - Expected Profit of $12,000

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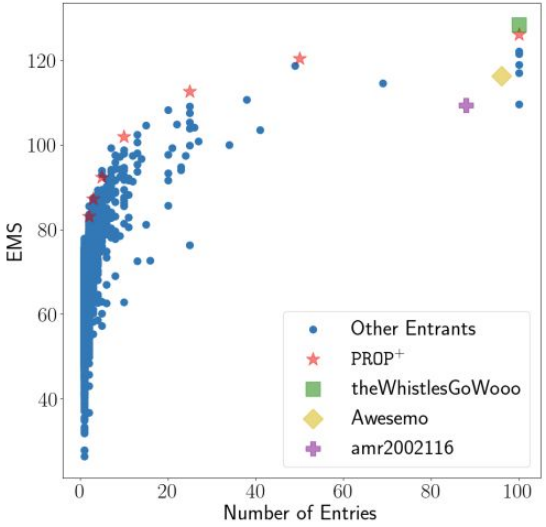
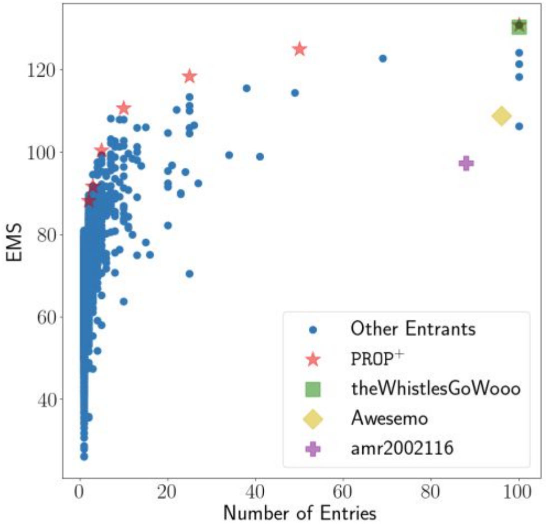
- Jeff Sonas, Ryan Holbrook, Addison Howard, Anju Kandru. (2024). March Machine Learning Mania 2024. Kaggle. https://kaggle.com/competitions/march-machine-learning-mania-2024/leaderboard

# DraftKings Contest (Robustness Check)




(a) Empirical EMS (cbbdata)




(b) Empirical EMS (seed-based)

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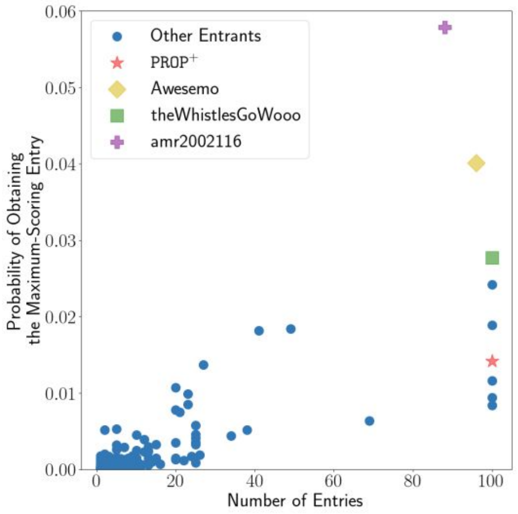


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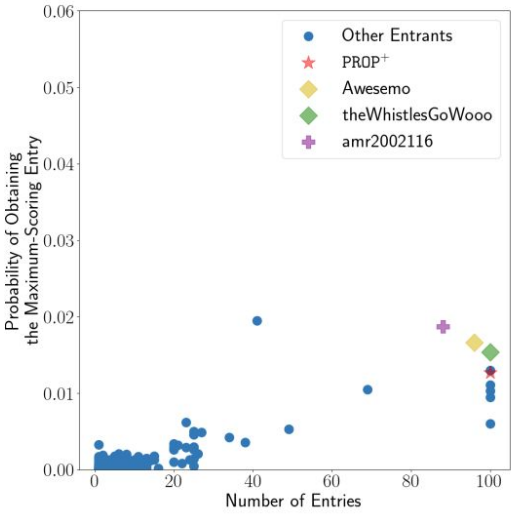
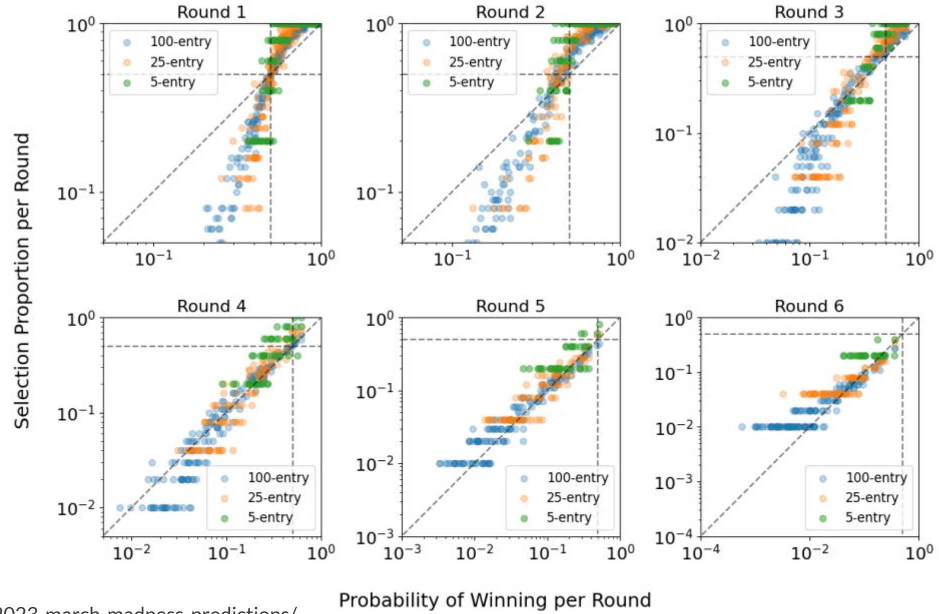



(b) Victory Probability (seed-based)

# Optimal Expected Score

## Prop+

| TEAM | REGION | POWER RATING | 1ST | 2ND | SWEET 16 | ELITE EIGHT | FINAL FOUR | CHAMP. | WIN |
|---|---|---|---|---|---|---|---|---|---|
| **Houston** 1 | Midwest | 93.2 | ✓ | 97% | 74% | 59% | 41% | 31% | 22% |
| **Alabama** 1 | South | 92.6 | ✓ | 99% | 82% | 65% | 45% | 30% | 16% |
| **Texas** 2 | Midwest | 90.1 | ✓ | 92% | 65% | 45% | 22% | 14% | 8% |
| **Purdue** 1 | East | 89.5 | ✓ | 98% | 69% | 41% | 25% | 12% | 5% |
| **Kansas** 1 | West | 89.6 | ✓ | 98% | 66% | 39% | 20% | 9% | 5% |
| **Gonzaga** 3 | West | 89.9 | ✓ | 92% | 64% | 38% | 22% | 9% | 5% |
| **Arizona** 2 | South | 89.0 | ✓ | 94% | 67% | 35% | 15% | 8% | 4% |
| **UCLA** 2 | West | 88.3 | ✓ | 95% | 70% | 37% | 21% | 8% | 3% |
| **UConn** 4 | West | 89.2 | ✓ | 85% | 65% | 31% | 15% | 6% | 3% |
| **Marquette** 2 | East | 87.6 | ✓ | 89% | 58% | 34% | 16% | 7% | 3% |
| **Baylor** 3 | South | 87.1 | ✓ | 89% | 45% | 24% | 10% | 6% | 3% |
| **Creighton** 6 | South | 87.6 | ✓ | 79% | 46% | 26% | 11% | 6% | 2% |
| **Duke** 5 | East | 87.1 | ✓ | 82% | 46% | 23% | 13% | 5% | 2% |
| **Tennessee** 4 | East | 86.9 | ✓ | 87% | 46% | 22% | 12% | 5% | 2% |
| **Kentucky** 6 | East | 86.0 | ✓ | 66% | 45% | 23% | 10% | 4% | 1% |
| **Indiana** 4 | Midwest | 85.8 | ✓ | 74% | 47% | 15% | 7% | 3% | 1% |
| **Texas A&M** 7 | Midwest | 85.1 | ✓ | 60% | 22% | 12% | 5% | 2% | 1% |
| **TCU** 6 | West | 85.4 | ✓ | 68% | 26% | 12% | 5% | 2% | 1% |
| **SDSU** 5 | South | 86.0 | ✓ | 67% | 39% | 10% | 5% | 2% | 0.8% |




10 Nate Silver (2023). 2023 March Madness Predictions. URL: https://projects.fivethirtyeight.com/2023-march-madness-predictions/